RESEARCH ARTICLE

Changing of Transport of Charge Carriers in an Implanted-junction Heterorectifier under the influence of Mismatch-induced Stress

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ABSTRACT: In this paper we analyzed changing of charge carrier's mobility in an implanted-junction heterorectifier under the influence of mismatch-induced stress. We introduce a model to describe the above changing of the mobility. Analysis of the above model leads to conditions (*i*) to change of the above mobility under influence of the considered stress; (*ii*) to change of mismatch- induced stress by radiation processing of materials of the considered multilayer structure. We also introduce an analytical approach to analyze of mass transport.

KEYWORDS: charge carriers mobility; implanted-junction rectifier; mismatch-induced stress; analytical approach for prognosis.

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INTRODUCTION

Currently one can find an intensive development of solid-state electronics devices. One of directions of the development is increasing of switching frequency of *p-n*- junctions [1-3]. To solve this problem, one can search new materials with higher values of charge carrier's mobility [4-7]. Another way to decrease the switching time is increasing of sharpness of *p-n*- junction [8,9]. Earlier it was shown that manufacturing of *p-n*- junctions by diffusion or ion implantation in a heterostructure and optimization of annealing time of dopant and/or radiation defects make it possible to increase sharpness of the above *p-n*- junctions [10-12]. It is known that there are mismatch- induced stress in heterostructure due to difference between lattice constants in layers of considered heterostructure are presented [13,14]. To analyze the influence of mismatch- induced stress on the values of charge carrier's mobility in a *p-n*- junctions, we consider the following situation. Let us consider a two-layer heterostructure, which consist of a substrate and an epitaxial layer with known types of conductivity (see Fig. 1). A dopant has been implanted into the substrate through the epitaxial layer to generate in it required type of conductivity, which is opposite to type of conductivity of the substrate. In this case, conditions of implantation are selected so that the main part of the dopant would be in the substrate. Next, the radiation defects were annealed. The main objective of this article is to analysis the changing of charge carrier's mobility in the implanted-junction rectifier under the influence of mismatch- induced stress. An accompanying aim of this article is to introduce an analytical approach for the analysis of mass transport processes, which makes it possible to take into account the simultaneous change in the parameters of the process under consideration in space and time, as well as its nonlinearity.

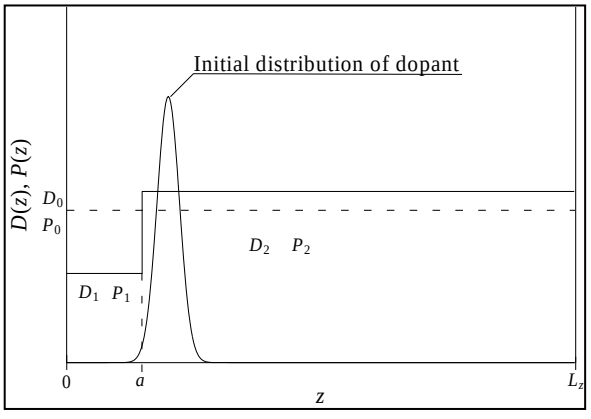

Fig. 1. Heterostructure, which consist of substrate and epitaxial layer

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Method of solution

To solve our aims let us calculate spatio-temporal distribution of dopant. We calculate the distribution by solution the second low of Fourier [1-3]

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_C\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_C\frac{\partial C(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_C\frac{\partial C(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+ \quad (1)$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]$$

with boundary and initial conditions

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$C(x,y,z,0)=f_C(x,y,z). \quad (2)$$

Here $C(x,y,z,t)$ is the spatiotemporal distribution of dopant; Ω is the atomic volume; symbol ∇_S denotes surface gradient; $\int_0^{L_z}C(x,y,z,t)dz$ is the surface concentration of dopant on interface between layers of heterostructure; $\mu(x,y,z,t)$ is the chemical potential; D_C and D_S are the coefficients of volumetric and surface diffusions, respectively. The surface diffusion in this situation is the effect of mismatch- induced stress. Values of the diffusion coefficients depend on properties of materials of layers of heterostructure, rates of heating and cooling of heterostructure spatio-temporal distributions of concentrations of dopant and radiation defects. The dependences can be approximated by the following relations [2]

$$D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$
$$D_S=D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \quad (3)$$

In the relations we used the following values: $D_L(x,y,z,T)$ and $D_{SL}(x,y,z,T)$ are the spatial (due to inhomogeneity of heterostructure and radiation damage of materials) and temperature (Arrhenius law) dependences of dopant diffusion coefficients; T is the annealing temperature; $P(x,y,z,T)$ is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer in the following interval $\gamma\in[1,3]$ [15]; $V(x,y,z,t)$ is the spatiotemporal distribution of vacancies; V^* is the equilibrium distribution of vacancies. Dependence of dopant diffusion coefficients on concentration of dopant are discussed in details in [15]. Dependences of dopant diffusion coefficients on concentrations of vacancies are generalization of analogous relation in [16]. The generalization is accounting for the generation of vacancies. The generation is accounting by quadratic terms of the approximation [18].

Spatiotemporal distributions of concentrations of radiation defects, we determine the solution for the following system of equations [17,18]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-$$

$$-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)\,V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right] \quad (4)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-$$

$$-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)\,V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]$$

with boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0, \quad (5)$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$I(x,y,z,0)=f_I(x,y,z)$, $V(x,y,z,0)=f_V(x,y,z)$.

Here $I(x,y,z,t)$ is the spatiotemporal distribution of interstitials; I^* is the equilibrium distribution of interstitials; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ are the volumetric and surface diffusion

coefficients of interstitials and vacancies; terms $V^2(x, y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of di-vacancies and di-interstitials (see, for example, [18] and appropriate references in the work); $k_{I,V}(x,y, z,T)$, $k_{I,I}(x,y,z,T)$ are $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point defects (first of them) and generation of their complexes, respectively.

We calculated the spatio-temporal distributions of concentrations of di-vacancies $\Phi_V(x, y,z,t)$ and di-interstitials $\Phi_I(x,y,z,t)$ as solution of the following system of equations [17-19]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right] + k_I(x,y,z,T)I(x,y,z,t) + k_{I,I}(x,y,z,T)\times$$

$$\times I^2(x,y,z,t) + \Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right] + \Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi IS}}{kT}\times\right.$$

$$\left.\times\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right] \qquad (6)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right] + k_V(x,y,z,T)V(x,y,z,t) + k_{V,V}(x,y,z,T)\times$$

$$\times V^2(x,y,z,t) + \Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right] + \Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi VS}}{kT}\times\right.$$

$$\left.\times\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=L_x} = 0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=0} = 0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=L_y} = 0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0} = 0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=L_z} = 0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=L_x} = 0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=0} = 0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=L_y} = 0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=0} = 0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z} = 0, \qquad (7)$$

$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z).$$

Here $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ are the diffusion coefficients of volumetric and surface diffusions of di-vacancies and di-interstitials, respectively; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of defects (di-vacancies and di-interstitials). The chemical potentials of equation (1), (4), (6) could be determined by the following relation [13,14]

$$\mu = E(z)\,\Omega\,\sigma_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2, \tag{8}$$

where $E(z)$ is the Young modulus, which depends on properties of layers of heterostructures; σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The Eq. (8) could be transform to the following form (Please check all the equation numbers once again)

$$\mu(x,y,z,t)=E(z)\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\}\frac{\Omega}{2},$$

where σ is Poisson coefficient; $\varepsilon_0 = (a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of substrate and epitaxial layers; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincide (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [20]

$$\begin{cases}\rho(z)\dfrac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\dfrac{\partial\sigma_{xx}(x,y,z,t)}{\partial x}+\dfrac{\partial\sigma_{xy}(x,y,z,t)}{\partial y}+\dfrac{\partial\sigma_{xz}(x,y,z,t)}{\partial z}\\ \rho(z)\dfrac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\dfrac{\partial\sigma_{yx}(x,y,z,t)}{\partial x}+\dfrac{\partial\sigma_{yy}(x,y,z,t)}{\partial y}+\dfrac{\partial\sigma_{yz}(x,y,z,t)}{\partial z}\\ \rho(z)\dfrac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\dfrac{\partial\sigma_{zx}(x,y,z,t)}{\partial x}+\dfrac{\partial\sigma_{zy}(x,y,z,t)}{\partial y}+\dfrac{\partial\sigma_{zz}(x,y,z,t)}{\partial z}\end{cases}$$

where

$$\sigma_{ij}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right]+K(z)\delta_{ij}\times$$

$$\times\frac{\partial u_k(x,y,z,t)}{\partial x_k}-\beta(z)K(z)\left[T(x,y,z,t)-T_r\right],\ \rho(z)$$ is the density of materials of considered heterostructure, δ_{ij} Is the Kronecker symbol. With account the relation for σ_{ij} last system of equation could be written as

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right]\times$$

$$\times\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$\times K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times$$

(9)

$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}-$$

$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Conditions for the system of Eq. (9) could be written in the form

$$\frac{\partial\vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(x,0,z,t)}{\partial y}=0;\ \frac{\partial\vec{u}(x,L_y,z,t)}{\partial y}=0;$$

$$\frac{\partial\vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial\vec{u}(x,y,L_z,t)}{\partial z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

First of all let us estimate the components of displacement vector. To make this procedure we used method of averaging of function corrections [11,12,19,21]. To determine the first-order approximations of required components of displacement vector we replace the required functions on their not yet known average values α_i. The substitution leads to the following results

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},$$

$$\rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2} = -K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},$$

$$\rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2} = -K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and the right sides of the above relations on time t lead to the following result

$$u_{1x}(x,y,z,t) = u_{0x} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta - K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1y}(x,y,z,t) = u_{0y} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta - K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1z}(x,y,z,t) = u_{0z} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta - K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined by using standard replacement of the required components in the right sides of Eqs. (9) on the following sums $\alpha_i+u_i(x,y,z,t)$ [11,12,19,21]. The replacement leads to the following result

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2} = \left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2} + \left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y} + \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right] - \frac{\partial T(x,y,z,t)}{\partial x}\times$$

$$\times K(z)\beta(z) + \left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2} = \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right] - \frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$\times K(z)\beta(z) + \frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z} + \frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\} + \frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\times$$

$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\,\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\,\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\,[1+\sigma(z)]}\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\,\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\,\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+$$

$$+\frac{E(z)}{6[1+\sigma(z)]}\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]-$$

$$\left.-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\Bigg\}\frac{E(z)}{1+\sigma(z)}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

.

Integration of the left and right sides of the above relations on time *t* leads to the following result

$$u_{2x}(x,y,z,t)=\frac{1}{\rho\,(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\Bigg\{K(z)-$$

$$-\frac{E(z)}{3\,[1+\sigma(z)]}\Bigg\}\frac{\partial^2}{\partial x\,\partial y}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\,\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\,\partial z}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\Bigg\{K(z)+$$

$$+\frac{E(z)}{3\,[1+\sigma(z)]}\Bigg\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\times$$

$$\times\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\,\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\tau\,d\vartheta\times$$

$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\,\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]-$$

$$-\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\,\tau\,d\,\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int_0^{t}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial x\,\partial y}\int_0^{t}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\times$$

$$\times\frac{1}{1+\sigma(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\,\partial y}\int_0^{t}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\times$$

$$\times\frac{\partial^2}{\partial y^2}\int_0^{t}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^{t}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial y}\int_0^{t}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^{t}\int_0^{\vartheta}T\,(x,y,z,\tau)d\,\tau\,d\,\vartheta-\left\{\frac{E(z)}{6\left[1+\sigma(z)\right]}-\right.$$

$$\left.-K(z)\right\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\,\partial z}\int_0^{t}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\,\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\frac{1}{1+\sigma(z)}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^{\infty}\int_0^{\vartheta}T\,(x,y,z,\tau)\,d\,\tau\,d\,\vartheta-\frac{K(z)}{\rho(z)}\times$$

$$\times\frac{\partial^2}{\partial x\,\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+\right.$$

$$\left.+K(z)\right\}-\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\right]\right\}\times$$

$$\times\frac{1}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\right]\frac{1}{\rho(z)}+\frac{1}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.\right.$$

$$+\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\Bigg[6\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\right.$$

$$\left.-\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\right\}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}.$$

Now we determine spatio-temporal distributions of concentrations of point radiation defects. To calculate the distributions we also used method of averaging of function corrections. Framework the approach we replace the concentrations I (x,y,z,t) and V (x,y,z,t) on their average values α_{1I} and α_{1V}. After the replacement we obtained equations for the first-order approximations of concentrations of point radiation defects I_1 (x,y,z,t) and V_1 (x,y,z,t)

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}\Omega L_z\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+\alpha_{1I}\Omega L_z\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]-$$

$$-\alpha_{1I}^2 k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,T)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}\Omega L_z\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+\alpha_{1V}\Omega L_z\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]-$$

$$-\alpha_{1V}^2 k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,T).$$

Integration on time of the left and right sides of the above equations gives us possibility to obtain the functions I_1 (x,y,z,t) and V_1 (x,y,z,t) in the final form

$$I_1(x,y,z,t)=\alpha_{1I}\Omega L_z\left[\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau\right]+$$

$$+f_I(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)d\tau \qquad (10)$$

$$V_1(x,y,z,t)=\alpha_{1V}\Omega L_z\left[\frac{\partial}{\partial x}\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau\right]+$$

$$+f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)d\tau.$$

Average values of the first-order approximations of concentrations of point radiation defects could be calculated by standard relation

$$\alpha_{1R}=\frac{1}{L_xL_yL_z\Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} R_1(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t, \qquad (11)$$

where $R = I,V$. Substitution of solutions (11) to the above relation gives us possibility to obtain the following equations for the average values α_{1I} and α_{1V}

$$\alpha_{1I} B_{I,I}^{1010} + \alpha_{1V} B_{I,V}^{1010} = \Omega A_{1I}^{0} + \Theta C_I - L_x L_y L_z \Theta$$

$$\alpha_{1I} B_{I,V}^{1010} + \alpha_{1V} B_{V,V}^{1010} = \Omega A_{1V}^{0} + \Theta C_V - L_x L_y L_z \Theta,$$

where

$$A_{iR}^{j} = \Omega \int_0^{\Theta} (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left(\frac{\partial}{\partial x} \left\{ \frac{D_{RS}}{kT} \nabla_S \mu\,(x,y,z,\tau) \int_0^{L_z} R_i^j (x,y,W,\tau)\, dW \right\} + \frac{\partial}{\partial y} \left\{ \frac{D_{IS}}{kT} \times \right.\right.$$

$$\left.\left. \times \frac{D_{IS}}{kT} \nabla_S \mu\,(x,y,z,\tau) \int_0^{L_z} R_i^j (x,y,W,\tau)\, dW \right\} \right) d\,z\, d\,y\, d\,x\, d\,t,$$

$$B_{R,R}^{ijkl} = \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{R,R}(x,y,z,T) \times$$

$$\times\, I_i^j (x,y,z,t) V_k^l (x,y,z,t) d\,z\, d\,y\, d\,x (\Theta - t)\, d\,t,\ C_R = \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_R (x,y,z) d\,z\, d\,y\, d\,x.$$

Solution of the system of equations is

$$\alpha_{1V} = \left[A_{1I}^{0} - L_x L_y L_z \Theta - \left(A_{1V}^{0} + C_V - L_x L_y L_z \Theta \right) \frac{B_{I,V}}{B_{V,V}} \right] \left(B_{V,V} + \frac{B_{I,V}^2}{B_{I,I}} \right),$$

$$\alpha_{1I} = \frac{A_{1I}^{0} + C_I - L_x L_y L_z \Theta - \alpha_{1V} B_{I,V}}{B_{I,I}}.$$

Approximations of the second and higher orders of concentrations of point radiation defects framework method of averaging of function corrections could be obtained by using standard iteration procedure, i.e. by replacing of the functions I (x,y,z,t) and V (x,y,z,t) on the following sums $\alpha_{nI} + I_{n-1}(x,y,z,t)$ and $\alpha_{nV} + V_{n-1}(x,y,z,t)$, where n is the order of approximation.

In the present paper we consider only the second-order approximations of the functions I (x,y,z,t) and V (x,y,z,t). The above substitution and integration on time of the left and right sides give us possibility to obtain the second-order approximations of concentrations of defects $I_2(x,y,z,t)$ and V_2 (x,y,z,t) in the following form

$$I_2(x,y,z,t) = \frac{\partial}{\partial x} \left[\int_0^t D_I (x,y,z,T) \frac{\partial I_1 (x,y,z,\tau)}{\partial x} d\,\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_I (x,y,z,T) \times \right.$$

$$\left. \times \frac{\partial I_1 (x,y,z,\tau)}{\partial y} d\,\tau \right] + \frac{\partial}{\partial z} \left[\int_0^t D_I (x,y,z,T) \frac{\partial I_1 (x,y,z,\tau)}{\partial z} d\,\tau \right] - \int_0^t k_{I,I} (x,y,z,T) \times$$

$$\times \left[\alpha_{2I} + I_1 (x,y,z,\tau) \right]^2 d\,\tau - \int_0^t k_{I,V} (x,T) \left[\alpha_{2I} + I_1 (x,y,z,\tau) \right] \left[\alpha_{2V} + V_1 (x,y,z,\tau) \right] d\,\tau +$$

$$+ f_I (x,y,z) + \Omega \frac{\partial}{\partial x} \left[\int_0^t \frac{D_{IS}}{kT} \nabla_S \mu\,(x,y,z,\tau) \int_0^{L_z} \left[\alpha_{2I} + I_1 (x,y,W,\tau) \right] dW\, d\,\tau \right] +$$

$$+\Omega\frac{\partial}{\partial y}\left[\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau\right]$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T)\times\right.$$

$$\left.\times\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{V,V}(x,y,z,T)\times$$

$$\times\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2d\tau-\int_0^t k_{I,V}(x,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+$$

$$+f_V(x,y,z)+\Omega\frac{\partial}{\partial x}\left[\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau\right].$$

Average values of the second-order approximation and approximations of the higher orders of concentrations of point radiation defects α_{nI} and α_{nV} could be calculated by using standard relation [11,12,19,21]

$$\alpha_{nR}=\frac{1}{L_xL_yL_z\Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[R_n(x,y,z,t)-R_{n-1}(x,y,z,t)\right]dz\,dy\,dx\,dt. \qquad (12)$$

Substitution of the first- and second-order approximations of the functions $I(x,y,z,t)$ and $V(x,y,z,t)$ into Eq. (12) gives us possibility to obtain equations for the average values α_{2I} and α_{2V}

$$\alpha_{2I}L_xL_yL_z\Theta=\Omega\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left(\frac{\partial}{\partial x}\left\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\right\}+\right.$$

$$\left.+\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\right\}\right)dz\,dy\,dx\,dt-\alpha_{1I}L_xL_yL_z\Theta-$$

$$-\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,V}(x,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]\left[\alpha_{2V}+V_1(x,y,z,t)\right]dz\,dy\,dx\,dt+C_I-$$

$$-\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]^2dz\,dy\,dx\,dt$$

$$\alpha_{2V}L_xL_yL_z\Theta=\Omega\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left(\frac{\partial}{\partial x}\left\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\right\}+\right.$$

$$+\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\right\}\Bigg)d\,z\,d\,y\,d\,x\,d\,t-\alpha_{1V}L_xL_yL_z\Theta-$$

$$-\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,V}(x,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]\left[\alpha_{2V}+V_1(x,y,z,t)\right]d\,z\,d\,y\,d\,x\,d\,t+C_V-$$

$$-\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,t)\right]^2d\,z\,d\,y\,d\,x\,d\,t.$$

Solution of the above system of equations could be written as

$$\alpha_{2I}=\frac{1}{2}\sqrt{\frac{1}{4}(a_6+a_2/a_1)^2-4\left[y+(a_2y-a_4)/a_1a_6\right]}-\frac{1}{4}(a_6+a_2/a_1),$$

$$\alpha_{2V}=\left[C_I-\alpha_{2I}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}\right)-B_{I,I}^{1210}-B_{I,V}^{1111}-\alpha_{1I}L_xL_yL_z\Theta+\right.$$

$$\left.+A_{1I}^1-\alpha_{2I}^2B_{I,I}^{1010}\right]/\left(\alpha_{2I}B_{I,V}^{1010}+B_{I,V}^{1110}\right),$$

where

$$a_1=B_{V,V}^{1010}\left(B_{I,I}^{1010}\right)^2-B_{I,I}^{1010}\left(B_{I,V}^{1010}\right)^2,\ a_2=B_{I,I}^{1010}B_{V,V}^{1010}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1V}^0+B_{I,V}^{1011}\right)-$$

$$-\left(B_{I,V}^{1010}\right)^2\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}\right)-B_{I,V}^{1010}\left(2B_{V,V}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}+B_{I,V}^{1110}\right)\times$$

$$\times B_{I,I}^{1010}+B_{I,V}^{1110},$$

$$a_3=B_{V,V}^{1010}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}\right)^2+\left(B_{I,V}^{1010}\right)^2\left(C_I-B_{I,I}^{1210}-B_{I,V}^{1111}+\right.$$

$$\left.+A_{1I}^1-\alpha_{1I}L_xL_yL_z\Theta\right)-\left(2B_{V,V}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}+B_{I,V}^{1110}\right)\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+\right.$$

$$\left.+B_{I,V}^{1011}\right)B_{I,V}^{1010}-B_{I,I}^{1010}B_{I,V}^{1110}\left(2B_{V,V}^{1110}+L_xL_yL_z\Theta-A_{1V}^0+B_{I,V}^{1011}\right)+B_{I,V}^{1111}-A_{1V}^1+\alpha_{1V}L_xL_yL_z\Theta+$$

$$+B_{V,V}^{1210}-C_V+2B_{I,V}^{1110}B_{I,V}^{1010}B_{I,V}^{1110},$$

$$a_4=B_{I,V}^{1010}\left(2B_{V,V}^{1110}+L_xL_yL_z\Theta-A_{1V}^0+B_{I,V}^{1011}+B_{I,V}^{1110}\right)\left(C_I+\right.$$

$$\left.+A_{1I}^1-B_{I,I}^{1210}-B_{I,V}^{1111}-\alpha_{1I}L_xL_yL_z\Theta\right)-B_{V,V}^{1010}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}\right)\left(C_I-B_{I,I}^{1210}-\right.$$

$$\left.-B_{I,V}^{1111}+A_{1I}^1-\alpha_{1I}L_xL_yL_z\Theta\right)-B_{I,V}^{1110}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta-A_{1I}^0+B_{I,V}^{1011}\right)\left(2B_{V,V}^{1110}+L_xL_yL_z\Theta-\right.$$

$$-A_{1V}^{0}+B_{I,V}^{1011}\Big)+2B_{I,V}^{1010}B_{I,V}^{1110}\Big(B_{I,V}^{1111}+\alpha_{1V}L_xL_yL_z\Theta-A_{1V}^{1}+B_{V,V}^{1210}-C_V\Big)+B_{I,V}^{1110}\Big(B_{I,V}^{1110}\Big)^2,$$

$$a_5=$$

$$=B_{V,V}^{1010}\Big(C_I-B_{I,I}^{1210}-B_{I,V}^{1111}+A_{1I}^{1}-\alpha_{1I}L_xL_yL_z\Theta\Big)^2+B_{V,V}^{1010}B_{I,I}^{1010}\Big(\alpha_{1I}L_xL_yL_z\Theta+B_{I,I}^{1210}+B_{I,V}^{1111}-$$

$$-C_I-A_{1I}^{1}\Big)+\Big(2B_{V,V}^{1110}+L_xL_yL_z\Theta-A_{1V}^{0}+B_{I,V}^{1011}\Big)\Big(C_I-B_{I,I}^{1210}-B_{I,V}^{1111}+A_{1I}^{1}-\alpha_{1I}L_xL_yL_z\Theta\Big)\times$$

$$\times B_{I,V}^{1110}+\Big(B_{I,V}^{1110}\Big)^2\Big(B_{I,V}^{1111}-A_{1V}^{1}+\alpha_{1V}L_xL_yL_z\Theta+B_{V,V}^{1210}-C_V\Big),\qquad a_6=-\sqrt{8y+\frac{a_2^2}{a_1^2}-4\frac{a_3}{a_1}},$$

$$q=\frac{a_3^3}{216a_1^3}-\qquad -a_3\frac{a_2a_4-4a_5}{48a_1^2}+\frac{a_4}{16a_1},\qquad y=\sqrt[3]{-q+\sqrt{q^2+p^3}}\ -\sqrt[3]{q+\sqrt{q^2+p^3}},$$

$$p=\frac{24a_1a_3-a_2^2}{13824a_1^2}.$$

System of Eqs. (6), we solve by using method of averaging of function corrections. Framework the approach we replace the functions Φ_I (x,y,z,t) and Φ_V (x,y,z,t) in right sides of Eqs. (6), on their average values $\alpha_{1\Phi I}$ and $\alpha_{1\Phi V}$. After the replacement we obtain the equations for the first-order approximations of concentrations of simplest complexes of radiation defects Φ_{1I} (x,y,z,t) and Φ_{1V} (x,y,z,t) in the following form

$$\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial t}=\alpha_{1\Phi I}\Omega L_z\frac{\partial}{\partial x}\left[\frac{D_{\Phi IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]\times$$

$$\times\alpha_{1\Phi I}\Omega L_z-k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$

$$\frac{\partial\Phi_{V1}(x,y,z,t)}{\partial t}=\alpha_{1\Phi V}\Omega L_z\frac{\partial}{\partial x}\left[\frac{D_{\Phi VS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi VS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]\times$$

$$\times\alpha_{1\Phi V}\Omega L_z-k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

Integration of left and right parts of the equations on time gives us possibility to obtain the first-order approximations of concentrations of di-vacancies and di-interstitials in the final form

$$\Phi_{I1}(x,y,z,t)=\alpha_{1\Phi I}\Omega L_z\frac{\partial}{\partial x}\left[\int_0^t\frac{D_{\Phi IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau\right]-k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\alpha_{1\Phi I}\Omega L_z\frac{\partial}{\partial y}\left[\int_0^t\frac{D_{\Phi IS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_{\Phi I}(x,y,z)$$

$$\Phi_{V1}(x,y,z,t)=\alpha_{1\Phi V}\Omega L_z\frac{\partial}{\partial x}\left[\int_0^t\frac{D_{\Phi VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\tau\right]-k_V(x,y,z,T)V(x,y,z,t)+$$

(13)

$$+\alpha_{1\Phi V}\Omega L_z\frac{\partial}{\partial y}\left[\int_0^t\frac{D_{\Phi VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)d\,\tau\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+\,f_{\Phi V}(x,y,z)$$

.

Average values $\alpha_{1\Phi I}$ and $\alpha_{1\Phi V}$ of the first-order approximations of concentrations of di-vacancies and di-interstitials could be calculated by standard relation, which analogous to relations (11) and (12). By calculation we obtain the following result

$$\alpha_{1\Phi I}=\left(\Theta F_I-E_I^1+E_I^2\right)\Big/L_z\left(L_xL_y\Theta-D_{1I}^0\right),$$
$$\alpha_{1\Phi V}=\left(\Theta F_V-E_V^1+E_V^2\right)\Big/L_z\left(L_xL_y\Theta-D_{1V}^0\right),$$

where $D_{iR}^j=\Omega\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left(\frac{\partial}{\partial x}\left\{\frac{D_{\Phi RS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\Phi_{Ii}^j(x,y,W,\tau)dW\right\}+\frac{\partial}{\partial y}\left\{\frac{D_{\Phi RS}}{kT}\times\right.\right.$

$$\left.\left.\times\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\Phi_{Ii}^j(x,y,W,\tau)\,dW\right\}\right)d\,z\,d\,y\,d\,x(\Theta-t)d\,t\,,$$

$$E_R^1=\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_R(x,y,z,T)\times$$

$$\times R(x,y,z,t)d\,z\,d\,y\,d\,x(\Theta-t)d\,t\,,$$

$$E_R^2=\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{R,R}(x,y,z,T)R^2(x,y,z,t)d\,z\,d\,y\,d\,x\times$$

$$\times(\Theta-t)\,d\,t\,,\ F_R=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_{\Phi R}(x,y,z)d\,z\,d\,y\,d\,x\,.$$

The second-order approximations of concentrations of di-vacancies and di-interstitials could be calculated by standard iteration procedure of method of averaging of function corrections. The approximations could be written as

$$\Phi_{I2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial x}d\,\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial y}\times\right.$$

$$\left.\times D_{\Phi I}(x,y,z,T)\,d\,\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial z}d\,\tau\right]+\int_0^t k_I(x,y,z,T)\times$$

$$\times I(x,y,z,\tau)d\,\tau+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi IS}}{kT}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi I}+\Phi_{2I}(x,y,W,\tau)\right]dW\,d\,\tau\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi IS}}{kT}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi I}+\Phi_{2I}(x,y,W,\tau)\right]dW\,d\,\tau\right\}+\int_0^t k_{I,I}(x,y,z,T)\times$$

$$\times I^2(x,y,z,\tau)\,d\,\tau+\,f_{\Phi I}(x,y,z)$$

$$\Phi_{V2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t \frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial y}\times\right.$$

$$\left.\times D_{\Phi V}(x,y,z,T)\,d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_V(x,y,z,T)\times$$

$$\times V(x,y,z,\tau)d\tau+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi VS}}{kT}\int_0^t \nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi V}+\Phi_{2V}(x,y,W,\tau)\right]dW\,d\tau\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi VS}}{kT}\int_0^t \nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi V}+\Phi_{2V}(x,y,W,\tau)\right]dW\,d\tau\right\}+\int_0^t k_{V,V}(x,y,z,T)\times$$

$$\times V^2(x,y,z,\tau)\,d\tau+f_{\Phi V}(x,y,z).$$

Average values $\alpha_{2\Phi I}$ and $\alpha_{2\Phi V}$ could be determined by standard relation. The relation is similar with relations (11) and (12). Results of calculation of the average values could be written as

$$\alpha_{2\Phi I}=\frac{D_{1I}^1+G_I^1+F_{RI}+E_I^2+E_I^1-\alpha_{1\Phi I}L_xL_yL_z\Theta}{L_xL_yL_z\Theta-D_{1I}^0},$$

$$\alpha_{2\Phi V}=\frac{D_{1V}^1+G_V^1+F_{RV}+E_V^2+E_V^1-\alpha_{1\Phi V}L_xL_yL_z\Theta}{L_xL_yL_z\Theta-D_{1V}^0},$$

where

$$G_R^i=\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left\{\frac{\partial}{\partial x}\left[D_{\Phi R}(x,y,z,T)\frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi R}(x,y,z,T)+\right.\right.$$

$$\left.\left.\times\frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi R}(x,y,z,T)\frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial z}\right]\right\}dz\,dy\,dx\,dt.$$

Distribution of dopant concentration we calculate by the same approach, which was used to calculate distributions of concentrations of defects. The first-order approximation of dopant concentration could be written as

$$C_1(x,y,z,t)=\alpha_{1C}L_z\Omega\frac{\partial}{\partial x}\left\{\int_0^t D_{SL}(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times\right.$$

$$\left.\times\frac{\nabla_S\mu\,(x,y,z,\tau)}{kT}\left[1+\frac{\alpha_{1C}^\gamma\xi_S}{P^\gamma(x,y,z,T)}\right]d\tau\right\}+\alpha_{1C}\frac{\partial}{\partial y}\left\{\int_0^t D_{SL}(x,y,z,T)\frac{\nabla_S\mu\,(x,y,z,\tau)}{kT}\times\right.$$

$$\left.\times\left[1+\frac{\alpha_{1C}^\gamma\xi_S}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]d\tau\right\}L_z\Omega+f_C(x,y,z).$$

Average value α_{1C} of the first-order approximation of dopant concentration $C_1(x,y,z,t)$ could be calculated by using standard relation, which is analogous to relations (11) and (12). Substitution of the

function $C_1(x,y,z,t)$ into appropriate relation we obtain equation for the average value in the final form α_{1C}

$$\alpha_{1C} = \alpha_{1C} H_1^0 + C_C$$

where

$$H_i^j = \Omega \int_0^{\Theta} (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[\frac{\partial}{\partial x} \left(D_{SL}(x,y,z,T) \left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \times \right. \right.$$

$$\times \frac{\nabla_S \mu\,(x,y,z,t)}{kT} \left\{ 1 + \xi_S \frac{\left[\alpha_{iC} + C_{i-1}^j(x,y,z,t)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} \Bigg) + \frac{\partial}{\partial y} \left(\left\{ 1 + \xi_S \frac{\left[\alpha_{iC} + C_{i-1}^j(x,y,z,t)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} \times \right.$$

$$\left. \left. \left. \times D_{SL}(x,y,z,T) \frac{\nabla_S \mu\,(x,y,z,t)}{kT} \left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \right) \right] d\,z\,d\,y\,d\,x\,d\,t$$

.

Solution of the equation depends on value of parameter γ.

The second-order approximation of dopant concentration $C_2(x,y,z,t)$ we determine framework the same approach, which has been used for calculation the second-order approximations of concentrations of radiation defects. After simple mathematical transformation we obtain the function $C_2(x,y,z,t)$ in the final form

$$C_2(x,y,z,t) = \frac{\partial}{\partial x} \left(\int_0^t \left\{ 1 + \xi \frac{\left[\alpha_{2C} + C_1(x,y,z,\tau)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \times \right.$$

$$\times D_L(x,y,z,T) \frac{\partial C_1(x,y,z,\tau)}{\partial x} d\,\tau \Bigg) + \frac{\partial}{\partial y} \left(\int_0^t D_L(x,y,z,T) \left\{ 1 + \xi \frac{\left[\alpha_{2C} + C_1(x,y,z,\tau)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} \times \right.$$

$$\left. \times \frac{\partial C_1(x,y,z,\tau)}{\partial y} \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \right) d\,\tau + \frac{\partial}{\partial z} \left(\int_0^t \frac{\partial C_1(x,y,z,\tau)}{\partial z} \times \right.$$

$$\times D_L(x,y,z,T) \left\{ 1 + \xi \frac{\left[\alpha_{2C} + C_1(x,y,z,\tau)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} \left[\varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} + \right.$$

$$\left. + 1 \right] d\,\tau \Bigg) + \frac{\partial}{\partial x} \left\{ \int_0^t \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \frac{\nabla_S \mu\,(x,y,z,\tau)}{kT} \times \right.$$

$$\left. \times \left\{ 1 + \xi \frac{\left[\alpha_{2C} + C_1(x,y,z,\tau)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)} \right\} d\,\tau \right) + \Omega \frac{\partial}{\partial x} \left\{ \int_0^t D_{SL}(x,y,z,T) \frac{\nabla_S \mu\,(x,y,z,\tau)}{kT} \times \right.$$

$$\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left[1+\xi_S\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right]\int_0^{L_z}[\alpha_{2C}+$$

$$+C_1(x,y,W,\tau)]dW\}d\tau+\Omega\frac{\partial}{\partial x}\left\{\int_0^t D_{SL}(x,y,z,T)\int_0^{L_z}[\alpha_{2C}+C_1(x,y,W,\tau)]dW\times\right.$$

$$\times\left[1+\xi_S\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\left.\times\frac{\nabla_S\mu(x,y,z,\tau)}{kT}d\tau\right\}+f_C(x,y,z).$$

We determine the parameter α_{2C} by standard relation, which analogous to relations (11) and (12). Substitution of the second-order approximation of dopant concentration $C_2(x,y,z,t)$ into appropriate relation and simple mathematical transformations leads to the following equation for the parameter α_{2C}

$$\alpha_{2C}=\alpha_{1C}+\frac{J_2^1+H_2^1+F_C}{L_xL_yL_z\Theta},$$

where

$$J_i^j=\Omega\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}D_L(x,y,z,T)\left[\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{[\alpha_{iC}+C_{i-1}(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\left[\varsigma_1\frac{V(x,y,z,t)}{V^*}+\right.\right.\right.$$

$$\left.\left.\left.+1+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_{i-1}(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{iC}+C_{i-1}(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.\right.$$

$$\left.\left.\times D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_{i-1}(x,y,z,t)}{\partial y}\right)\right]dz\,dy\,dx(\Theta-t)dt$$

.

Now let us analyze redistribution of dopant and radiation defects, as well as changes of components of displacement vector in the considered heterostructure. The obtained analytical relation gives a possibility to make analysis of the redistribution of both dopant and radiation defects, and changes in the components of the displacement vector during annealing. Using numerical approaches makes it possible to check the obtained analytical results. In this situation the analytical solutions of Equations (1), (4), (6) and (9) were verified by comparison with results of direct numerical simulation.

DISCUSSION

Let us consider the mobility of charge carriers with allowance for their scattering on neutral atoms of impurity. The corresponding relation was obtained in [22] and could be written as

$$\mu=\frac{e^2m^*}{20\,\varepsilon_r\hbar^3C(x,t)},\quad(14)$$

where e is the electron charge, m^* is the effective mass of the electron, ε_r is the relative dielectric constant of the material, $\hbar\approx1{,}054\cdot10^{-34}$ J·c is the Planck's constant. Substitution of the obtained distributions of the impurity concentration into relation (14) allows one to obtain the dependences of the mobility of

charge carriers on the coordinate and time. These dependences are shown in Fig. 2. Curve 1 in Fig. 2 corresponds to a material with compressive stress; curve 2 corresponds to an unstressed material. The opposite relationship between the curves occurs under tensile stresses. It should also be noted that the presence of radiation treatment of the heterostructure during ion doping makes it possible to reduce the value of mechanical stresses (see Fig. 3).

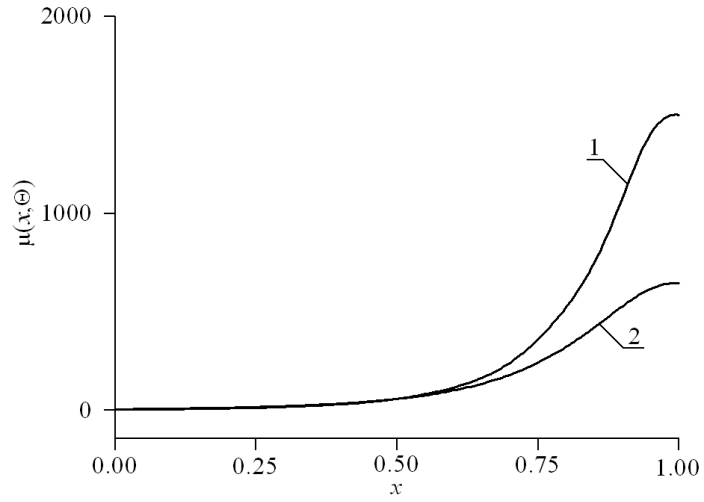

Fig. 2. Normalized distributions of the mobility of charge carriers in the considered heterostructure along the interface. Curve 1 corresponds to the compressive stress epitaxial layer. Curve 2 corresponds to an unstressed epitaxial layer

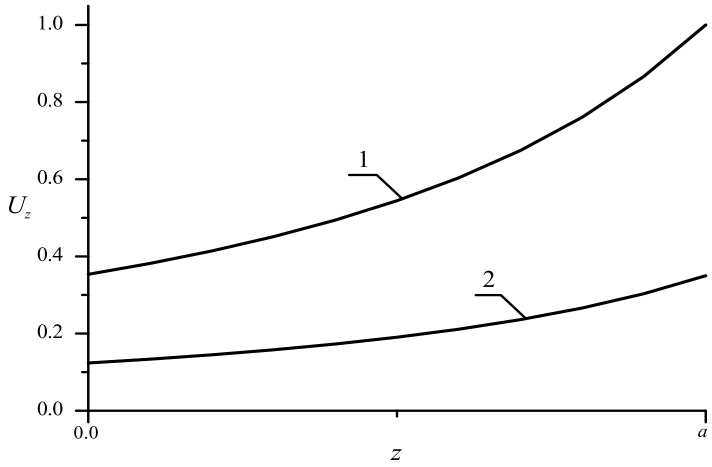

Fig. 3. Normalized dependences of the component of the displacement vector u_z on the coordinate z for unstressed (curve 1) and stressed (curve 2) epitaxial layers

CONCLUSION

In this paper we analyzed changing of charge carrier's mobility in an implanted-junction rectifier under influence of mismatch-induced stress. A model for description the considered situation is introduced. An analytical approach for analysis of mass transfer is introduced. The model gives a possibility to take into account the simultaneous changing of parameters of mass transport in space and time. At the same time the approach gives a possibility to take into account nonlinearity charge carrier's transport.

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